Behavioral Game Theory: Predicting Human Behavior in a normal-form game

Ruiyu Gou, CS 532L Feb 2023

Agenda

Motivation (5 mins)

Related Fields (2 mins)

Models (10 mins)

- Quantal Response Equilibrium
- Bounded Iterative Reasoning (Level-K & Cognitive Hierarchy)

Discussion (3 mins)

Agenda

Motivation

Related Fields

Models

Discussion

Traveler's Dilemma

Motivational Example

> "We know that the bags have identical contents, and we will entertain any claim between \$180 and \$300, but you will each be reimbursed at an amount that equals the *minimum* of the two claims submitted. If the two claims differ, we will also pay a reward *R* to the person making the smaller claim and we will deduct a penalty *R* from the reimbursement to the person making the larger claim."

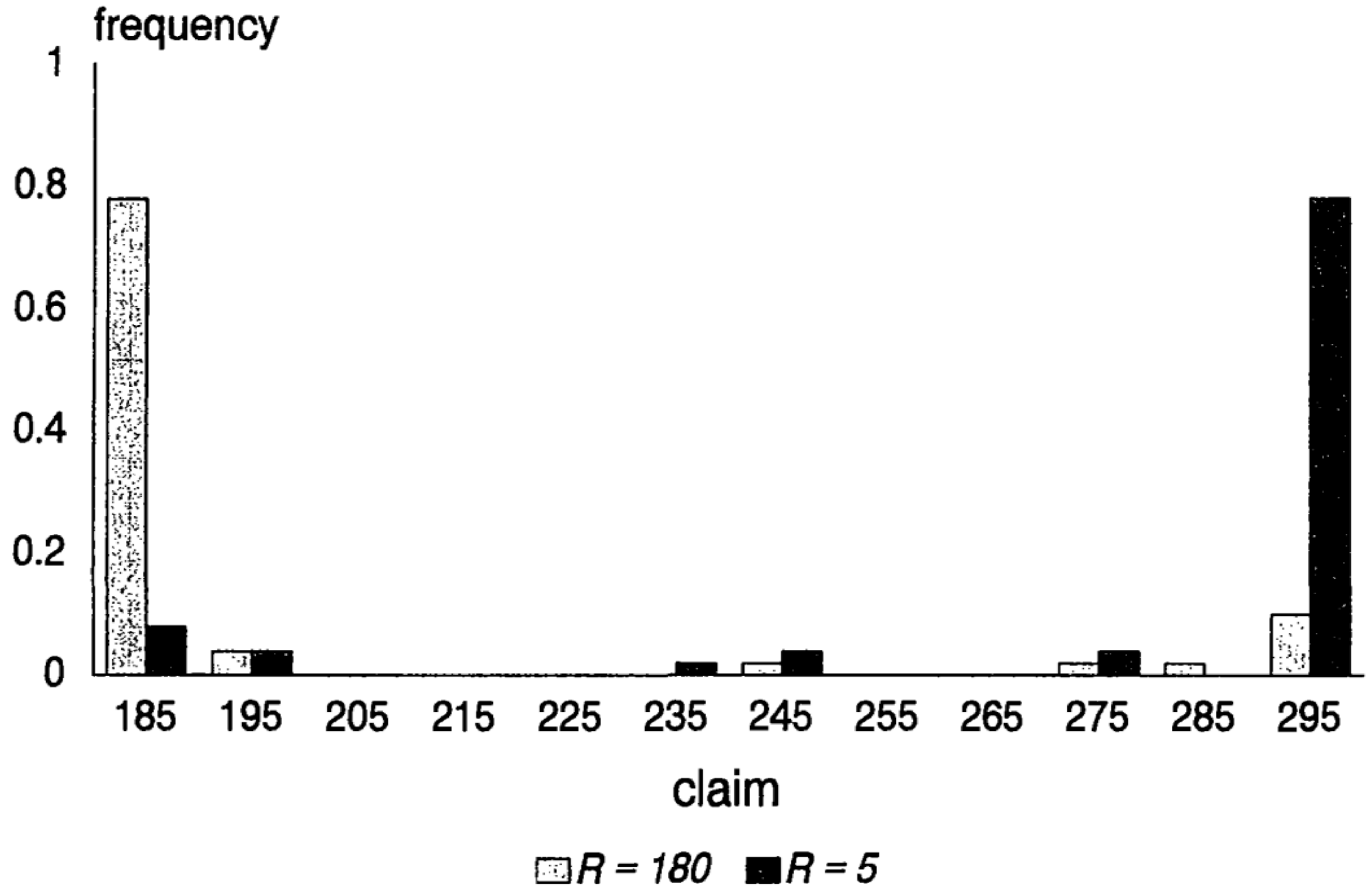
- **Experimental results deviate from NE**
    - R=5, 11/12 (91.7%) of the class did not play Nash.
    - We are so irrational! (except Sophie)

Experimental results deviates from NE

[Goeree, Holt, 2001]

[$180, $300], R=5, R=180

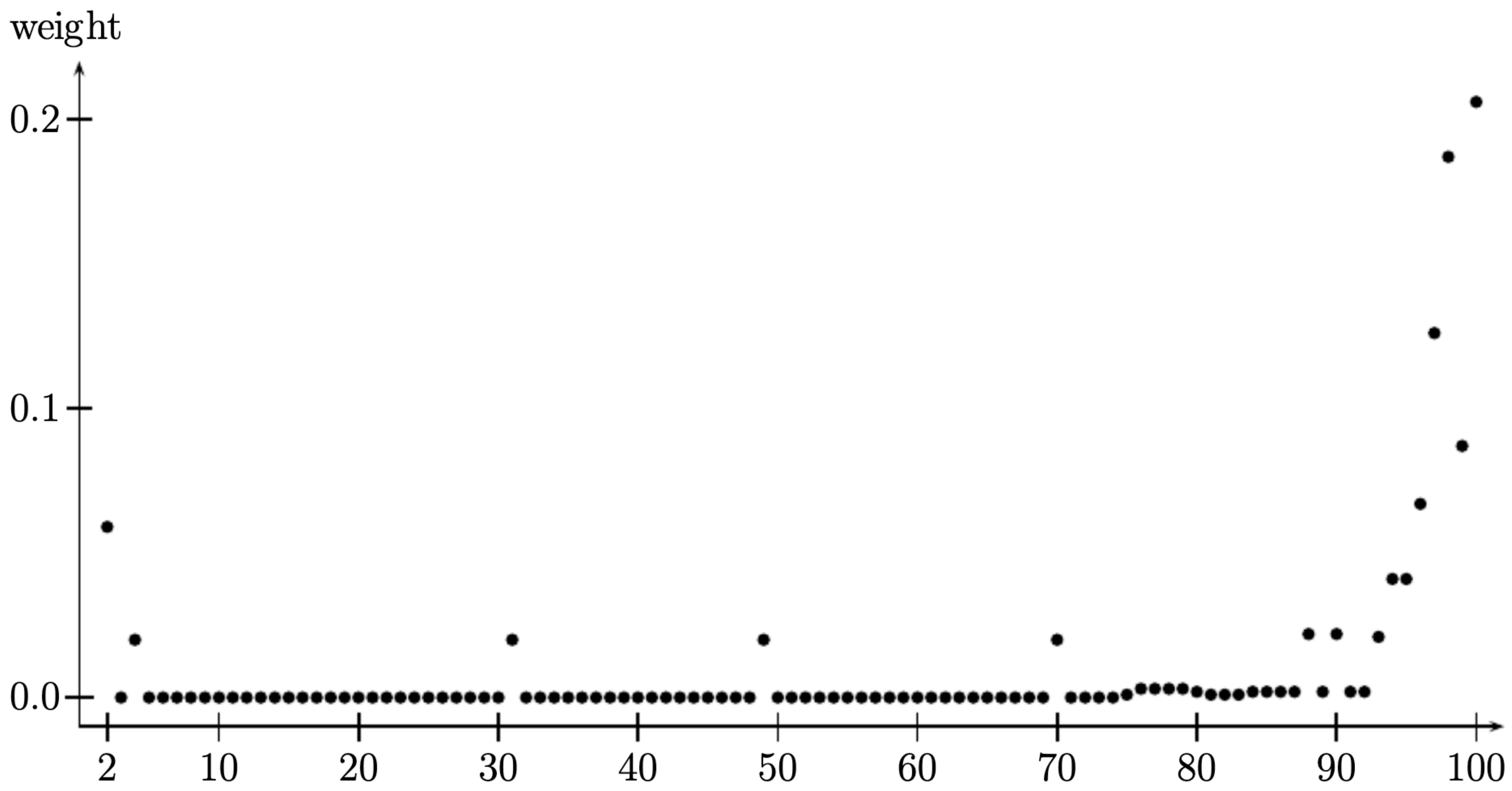
- 50 random subjects (25 pairs)
- ~80% claimed the highest amount $300, average claim $280



[Becker, et al. 2005]

[$2, $100], R=2

- 51 members of Game Theory Society
- ~20% played highest amount



Behavioral Game Theory (BGT) seeks to explain this deviation.

Agenda

Motivation

Related Fields

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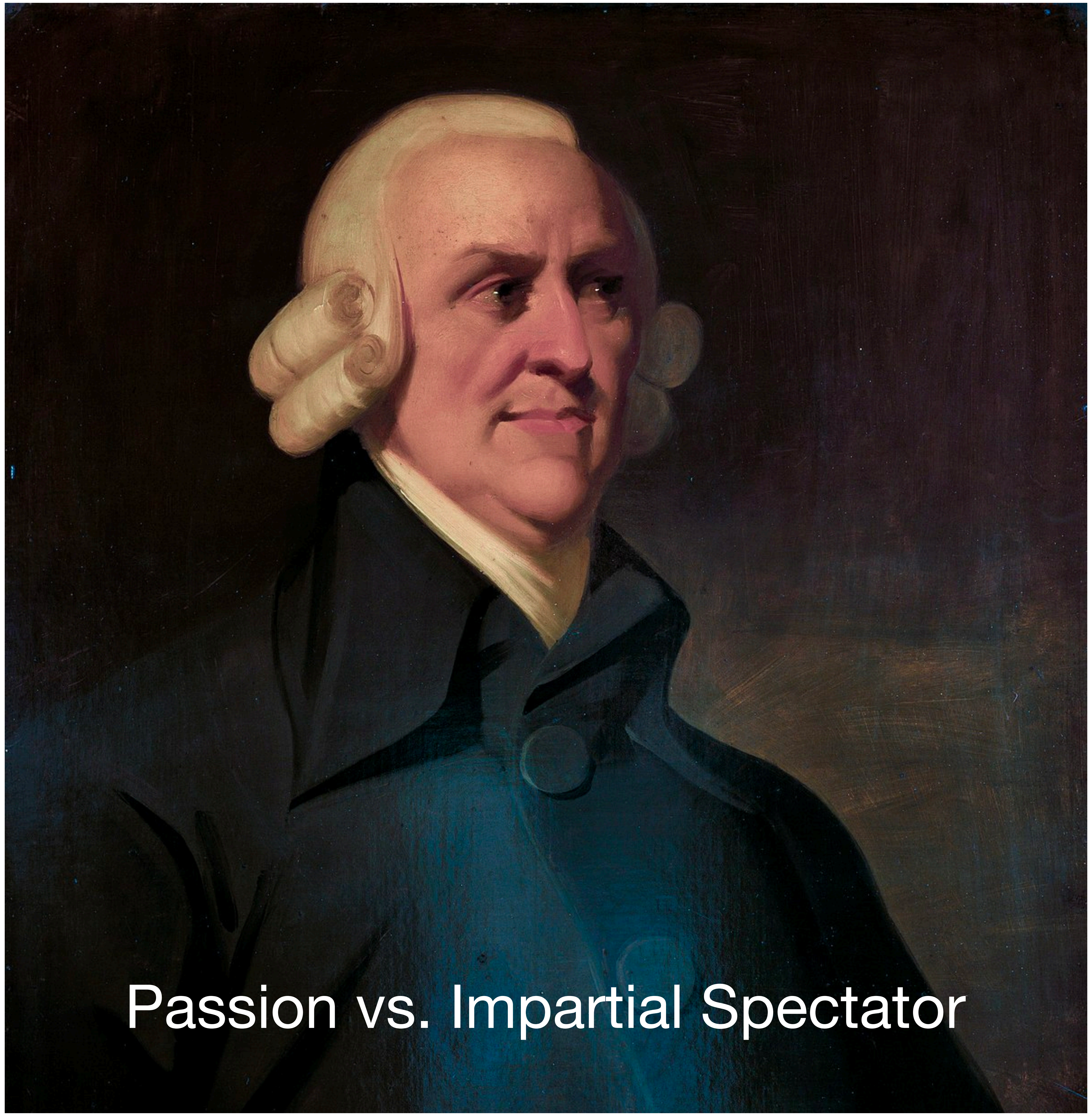
Discussion

Behavioural Economics

- concerned with bounded rationality of economic agents
- studies market decisions, public opinions
- Examples:
  - Loss aversion
  - Fairness
  - Discounted Utility



Psychology

- Methods
    - Experimental psychology
- Concepts
    - Emotions (fear, regret, shame etc.)
    - Deeper motivations (reciprocity, guilt)
- Complex and dynamic, hard to quantize into utility

BGT in unrepeated normal-form games

1994

Quantal level-k
(Stahl and Wilson, 1994)

Quantal Response Equilibrium
(McKelvey and Palfrey, 1995)

level-k
(Nagel, 1995;
Costa-Gomes et al., 2001)

2004

Cognitive Hierarchy
(Camerer et al. 2004)

Noisy Introspection
(Goeree and Holt, 2004)

...

Agenda

Motivation

Related Fields

Models:

Behavioral Model:
$P(a_i \mid G_i, \theta)$

Quantal Response Equilibrium

Bounded Iterative Reasoning (Level-K & Cognitive Hierarchy)

Discussion

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Models:

- Quantal Response Equilibrium
- Bounded Iterative Reasoning (Level-K & Cognitive Hierarchy)

Discussion

Quantal Response Equilibrium (QRE)

McKelvey and Palfrey, 1995

- Intuition:
  - Players can make errors, but less likely when error gets more costly.
- Key idea: maximizing expected utility with some noise

$$\hat{u}_i(a_i, s_{-i}) = \underbrace{u_i(a_i, s_{-i})}_{\text{true utility}} + \underbrace{\epsilon_{a_i}}_{\text{noise}}$$

Quantal Response Equilibrium (QRE)

Given $\hat{u}_i(a_i, s_{-i}) = \underbrace{u_i(a_i, s_{-i})}_{\text{true utility}} + \underbrace{\epsilon_{a_i}}_{\text{noise}}$

QRE is a strategy profile s^* where for every agent i :

$$\hat{u}_i(s^*) \in \arg\max_{s_i} \hat{u}(s_i, s^*_{-i})$$

Similar to NE, a **quantal response equilibrium** is a mixed strategy profile in which every agent's strategy is a **quantal best response** to the strategies of the other agents.

Logit Quantal Best Response

"precision": How sensitive agents are to utility differences

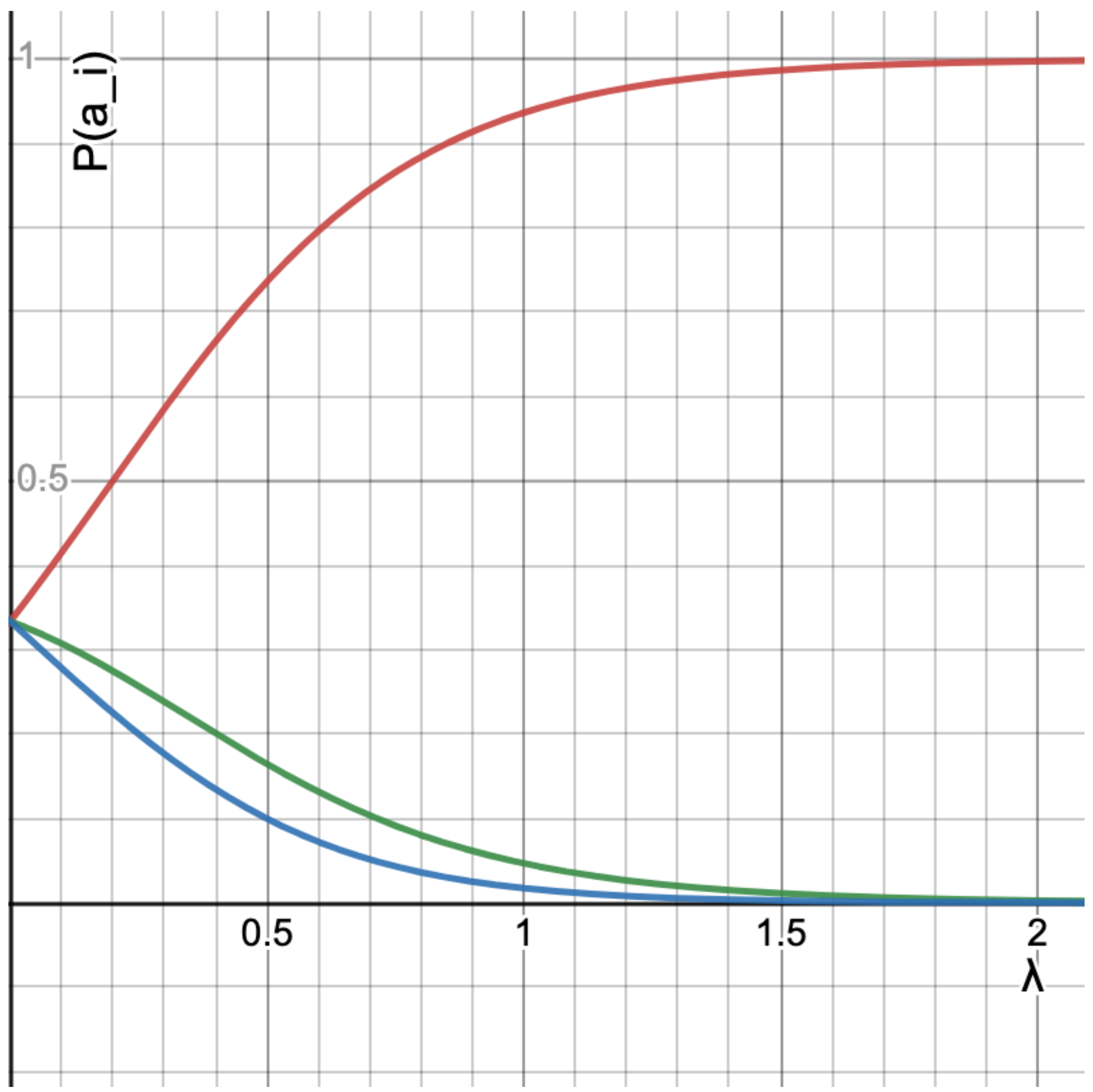
$$s_i^*(a_i) = \frac{e^{\lambda \cdot u_i(a_i, s_{-i}^*)}}{\sum_{a_i'} e^{\lambda \cdot u_i(a_i', s_{-i}^*)}}$$

$\lambda = 0$, Uniform Distribution

$\lambda \to +\infty$, Nash Equilibrium

Example:
One player,
3 action choices with utility [6, 3, 2]

<u>visualization of its action probabilities:</u>



Revisit Traveler's Dilemma
with QRE

- Experiments show dramatic shifting of claims with change of penalty.
- **Well tracked** by QRE.
- Noise can "snowball".

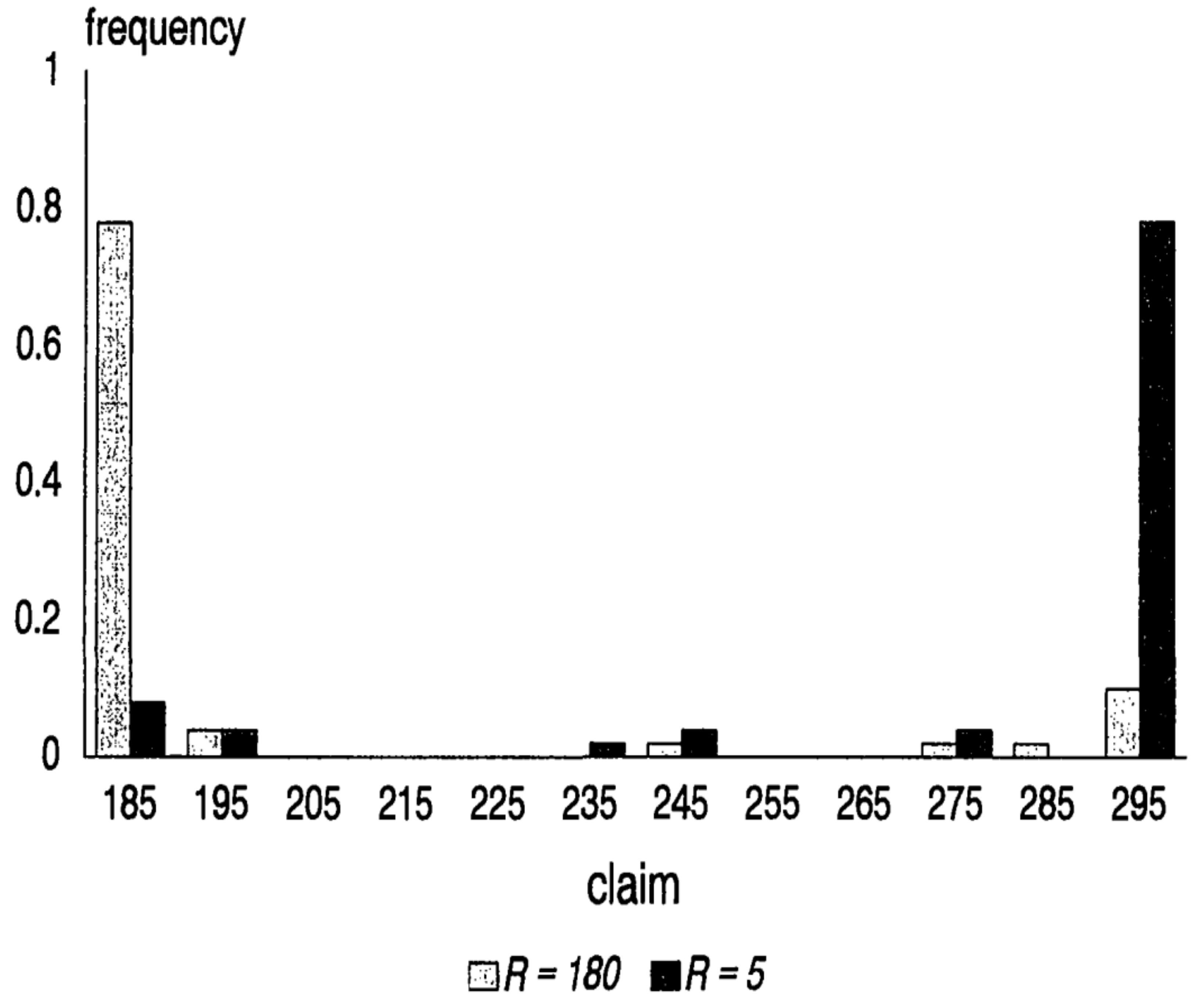


FIGURE 1. CLAIM FREQUENCIES IN A TRAVELER'S DILEMMA FOR R = 180 (LIGHT BARS) AND R = 5 (DARK BARS)

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- Bounded Iterative Reasoning (Level-K & Cognitive Hierarchy)

Discussion

Level-k Thinking

(Stahl and Wilson, 1995; Nagel, 1995)

- Each player assumes their strategy is the most **sophisticated** **(degree of recursion)**
- Inductively defined strategies:
  - step 0 players: randomize
  - step 1 players: best respond to step 0 players
  - …
  - step k players: best respond to step k-1 players

Cognitive Hierarchy

(Camerer et al. 2004)

- Each player assumes their strategy is the most sophisticated
- Inductively defined strategies:
    - step 0 players: randomize
    - step 1 players: best respond to step 0 players
    - …
    - step k players: best respond to players **distributed over step 0 to k-1**

Revisit Traveler's Dilemma
With Bounded Iterative Reasoning

- Most of us played $300, but some played differently
  - $180
  - $298?
  - $295?

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Discussion and Limitations

- The two paradigms often make similar and improved predictions for experimental results.
- Many models similar to their variants or a mixture of both.
- Subject to overfitting.
- Cognitive Hierarchy and Level-K assumed uniform strategies of level-0 agents; this does not seem plausible.

Summary

- Behavioural Game Theory is concerned with what **human** do in a game. They explain **experimental results** sometimes better than NE.
- **Quantal Response** introduces noises in action probabilities around best responses, **QRE** is the equilibrium where such responses are considered.
- **Cognitive Hierarchy and Level-K Thinking** assumed bounded depth of iterative reasoning, when players try to reason about what the other players think.
- Although the above models focus on explaining observations, recent development in BGT seeks to predict and generalize.

Paradox of Rationality

"Players who make irrational or naïve choices often receive better payoffs and that those making the rational choices predicted by backward induction often receive worse outcomes."

References

Primary sources (plus the ones on slides)

Ten Little Treasures of Game Theory and Ten Intuitive Contradictions, Goeree and Holt, 2001

Beyond Equilibrium: Predicting Human Behaviour in Normal Form Games, J.R. Wright 2010

Modeling Human Behavior in Strategic Settings, J.R. Wright, 2016

Predicting human behavior in unrepeated, simultaneous-move games, Wright and Leyton-Brown, 2017

A Cognitive Hierarchy Model of Games, Camerer, 2004

A Case for Behavioural Game Theory, Sarah Bonau, 2017

Noise distribution assumption for LQRE

In the rest of the paper, we study a particular parametric class of quantal response functions that has a tradition in the study of individual choice behavior (Luce, 1959). For any given $\lambda \geq 0$, the *logistic* quantal response function is defined, for $x_i \in \mathbb{R}^{J_i}$, by

$$\sigma_{ij}(x_i) = \frac{e^{\lambda x_{ij}}}{\sum_{k=1}^{J_i} e^{\lambda x_{ik}}}$$

with player i, action $j \in J_i$

$\hat{u}_{ij} = u_{ij} + \underline{\underline{\varepsilon_{ij}}}$

$\varepsilon_i = (\varepsilon_{i1}, \ldots \varepsilon_{iJ_i}) \sim$ distribution with PDF $f_i(\varepsilon_i)$

(marginal distribution of f_i exists for each ε_{ij} and $E(\varepsilon_i)=0$)

and corresponds to optimal choice behavior[4] if f_i has an extreme value distribution, with cumulative density function $F_i(\varepsilon_{ij}) = e^{-e^{-\lambda\varepsilon_{ij}-\gamma}}$ and the ε_{ij}'s are independent. Therefore, if each player uses a logistic quantal response function, the corresponding QRE or *Logit Equilibrium* requires, for each i, j,

$$\pi_{ij} = \frac{e^{\lambda x_{ij}}}{\sum_{k=1}^{J_i} e^{\lambda x_{ik}}}$$

where $x_{ij} = \bar{u}_{ij}(\pi)$.